AP Calculus BC

WS93 - Power Series Review

1) $\sum_{n=0}^{\infty}\left(-\frac{x}{3}\right)^n$

GST

$\left|-\frac{x}{3}\right| < 1$

$-1 < \frac{x}{3} < 1$

$-3 < x < 3$

$R = 3$

(D)

2) $\ln(1+x) = f(x)$

$f(0) = 0$

$f'(x) = \frac{1}{1+x}$ $f'(0) = 1$

$f''(x) = -(1+x)^{-2}$ $f''(0) = -1$

$f'''(x) = 2(1+x)^{-3}$ $f'''(0) = 2$

$M(x) = 0 + x - \frac{1}{2!}(x)^2 + \frac{2}{3!}(x)^3$

$= x - \frac{x^2}{2} + \frac{x^3}{3}$ (B)

3) $\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{n!} = e^{-x}$

(D)

4) $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(5n)!}$

Ratio Test

$\lim_{n\to\infty}\left|\frac{(-1)^{n+1}x^{2n+2}}{(5n+5)!}\cdot\frac{(5n)!}{(-1)^n x^{2n}}\right| = \lim_{n\to\infty}\frac{x^2\cdot(5n)!}{(5n+5)!}$

$= 0$

$R = \infty$ I: $(-\infty, \infty)$ (B)

5) $\frac{d}{dx}\left[\sum_{n=0}^{\infty}\frac{2x^n}{n!}\right]$

$= \sum_{n=1}^{\infty}\frac{2x^{n-1}}{(n-1)!} = 2e^x$

(E)

6) $e^x = 1 + x + \frac{x^2}{2} = A + Bx + Cx^2$

$A = 1$; $B = 1$; $C = \frac{1}{2}$

$A + B + C = \frac{5}{2}$ (C)

7) $f(x) = x^7 e^{x^5}$

$e^{x^5} = 1 + x^5 + \frac{x^{10}}{2} + \frac{x^{15}}{3!} + \cdots$

$x^7 e^{x^5} = x^7 + x^{12} + \frac{x^{17}}{2} + \frac{x^{22}}{3!} + \cdots$

$f^{(17)}(x) = \frac{17!}{2} + \frac{22!\,x^5}{5!\,3!} + \cdots$

$f^{(17)}(0) = \frac{17!}{2}$

(D)

8) $\sin x = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}$

(A)

9) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$

error $< 0.01 = \frac{1}{100}$

use $\left|\frac{1}{101}\right|$ $a_n = \frac{1}{2n-1}$

$2n - 1 = 101$

$2n = 102$

$\boxed{n = 51}$